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$N = 2$ Type I-Heterotic Duality and Higher Derivative F-Terms

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Abstract

We test the conjectured Type I-Heterotic Duality in four dimensions by analyzing a given class of higher derivative F-terms of the form $F_g W^{2g}$, with W the $N = 2$ gravitational superfield. We study a particular dual pair of theories, the $O(2,2)$ heterotic model and a type I model based on the $K3$ $\mathbf{Z_2}$ orbifold theory constructed by Gimon and Polchinski, further compactified on a torus. The F_g couplings appear at 1-loop on both theories; because of the weak-weak nature of this duality in four dimensions, it is meaningful to compare the heterotic F_g's with the corresponding type I couplings perturbatively. We compute the F_g's in type I, showing that they receive contributions only from $N = 2$ BPS states and that in the appropriate limit they coincide with the heterotic couplings, in agreement with the given duality.

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1. Introduction

One of the most important results achieved in the last two years, towards an understanding of non-perturbative string theory, has been the equivalence of string theories previously considered as different. One of these equivalences has been proposed by Polchinski and Witten in [1], where they gave some arguments leading to the conclusion that type I and heterotic theory are dual to each other. Although in ten dimensions this is a strong-weak duality, it turns out that after suitable compactifications leading to supersymmetric $N = 2$ theories in four dimensions, there exist regions in the moduli space of the theory in which both the heterotic and the type I descriptions are simultaneously weakly coupled.

In four dimensions $N = 2$ string theories present another duality, realized by dual pairs constructed from Calabi-Yau and $K3 \times T^2$ compactifications of the type II and heterotic string respectively [2]. While there have been several and strong checks for the aforementioned type II-heterotic duality [3, 4], only recently the study of the four dimensional consequences of type I-heterotic duality has been started in ref.[5]. They studied the 1-loop corrections to the prepotential for a class of models obtained by compactifying type I on $T^4/\mathbf{Z_2} \times T^2$ [6]. In particular it has been shown in [5] that the perturbative prepotential for a rank four model that admits type I, type II and heterotic descriptions, agree.

The aim of this note is to compute in type I, for the rank four model, 1-loop amplitudes arising from the higher derivative F-terms of the form $F_g(X)W^{2g}$ where W is the $N = 2$ gravitational superfield and X denotes generically the chiral vector superfields of the theory. They are then compared, in the appropriate limit, to the corresponding heterotic couplings that have been already computed in [3] for a $O(2, 1)$ model (and whose result has been extended in [7] for more general $O(2, n)$ models).

This note is organized as follows. In the next section we briefly review the construction of the type I-heterotic dual pair under consideration, looking for the appropriate limit in the vector moduli space in which the two 1-loop amplitudes have to coincide. In section

three we recall the form of the F_g couplings in the heterotic theory and then, in section four, we compute these terms in the type I side, showing that they are in complete agreement with the heterotic one's, providing a further check for the given duality.

2. The dual pair

In this section we shortly remind how to construct a specific type I-heterotic rank four dual pair. On the heterotic side it is obtained by compactifying the $E_8 \times E_8$ theory on $K3 \times T^2$, putting 12 instantons on each E_8, and then higgsing completely the remaining two E_7's. This leaves us with a massless spectrum containing 244 hypermultiplets and just the three vectormultiplets coming from the torus [2]. Note that since one vector-field is the graviphoton, the vector moduli space is parametrized by three complex scalars, usually denoted S-T-U, where

$$S_H = \theta + ie^{-2\phi_H}, \quad T_H = B_{45} + i\sqrt{G}, \quad U_H = (G_{45} + i\sqrt{G})/G_{44} \tag{2.1}$$

denote the complex dilaton, the Kähler and complex structure of the torus, respectively [1]. As already mentioned in the introduction, the type I dual is constructed starting from the six-dimensional models on a $K3$ orbifold $T^4/\mathbf{Z_2}$ studied in [6]. The closure of the operator product expansion and the cancellation of tadpole divergencies require the presence of 32 9-branes and 32 5-branes. Because of U(1) anomalies in the theory, putting two 5-branes at each fixed point and giving non-vanishing vacuum expectation values to scalars, it is possible to break completely the gauge group, remaining only with 244 $N = 1$ hypermultiplets in six dimensions [5, 8], 4 of which coming from the closed string sector and the remaining 240 from the open string sector. After the further compactification on T^2, the massless spectrum consists of 4 U(1)'s, 3 complex scalars and the 244 $N = 2$ hypermultiplets, of course. It is convenient in type I to take the following combinations of the three complex

[1]From now on a subscript H or I refers to heterotic and type I quantities.

scalars:

$$S_I = \theta + ie^{-\phi_I} G^{1/4}\omega^2, \quad S_I' = B_{45} + ie^{-\phi_I} G^{1/4}\omega^{-2}, \quad U_H = (G_{45} + i\sqrt{G})/G_{44} \qquad (2.2)$$

where ω^4 is the volume of the $K3$ orbifold. It is now straightforward to show that starting from the ten-dimensional relations $\phi_I^{10} = -\phi_H^{10}$ and $\alpha_I' = e^{\phi_H^{10}}\alpha_H'$ [1], the duality map becomes [5]

$$S_H = S_I, \quad T_H = S_I', \quad U_H = U_I \qquad (2.3)$$

So weakly coupled type I theory corresponds to weakly coupled heterotic theory provided that the volume of the T^2 torus is large. It is important to note that the type I low-energy effective action is actually invariant under the two Peccei-Quinn symmetries $Re\, S_I \to Re\, S_I$ + const., $Re\, S_I' \to Re\, S_I'$ + const., implying the perturbative independence of chiral amplitudes (in loops) from S_I and S_I'. On the other hand the heterotic effective action presents only the usual P-Q symmetry, $Re\, S_H \to Re\, S_H$ + const. Given the relations (2.3), this immediately implies that for every chiral amplitude A:

$$\lim_{T_2 \to \infty} A_H(T, U) = A_I(U)|_{S_2 > S_2'} \qquad (2.4)$$

where $T_2 = Im\, T$ [2].

3. Heterotic F_g couplings in the $T_2 \to \infty$ limit

We briefly review in this section the computation done in [3] of these couplings, in order to take the $T_2 \to \infty$ limit. The relevant amplitude for $F_{g,H}$ involves two gravitons and $2g-2$ graviphotons. It has been shown in [3] that these amplitudes contain only correlators of space-time bosons that can be easily computed by defining a generating function for the $F_{g,H}$'s [3], $F_H(\lambda, T, U) \equiv \sum_{g=1}^{\infty} g^2 \lambda^{2g} F_{g,H}(T, U)$. This allows the exponentiation of the

[2] The rescriction $S_2 > S_2'$ is due to the fact that in the heterotic theory the large S_H limit is taken before the T_H limit [5].

[3] This is slightly different from the generating function defined in [3], in order to compare better the heterotic results with the type I one's.

operators, reducing in this way the computation to the evaluation of a determinant. From the results given by [3], it is not difficult to show that

$$F_H(\lambda,T,U)=\frac{\lambda^2}{\pi^2}\int\frac{d^2\tau}{\tau_2}\frac{1}{\bar\eta^4(\bar q)}C(\bar q)\sum_{\substack{n_1,n_2\\m_1,m_2}}q^{\frac{1}{2}|P_L|^2}\bar q^{\frac{1}{2}|P_R|^2}\frac{1}{2}\frac{d^2}{d\tilde\lambda^2}\left[\left(\frac{2\pi i\tilde\lambda\bar\eta^3(\bar q)}{\bar\theta_1(\tilde\lambda,\bar\tau)}\right)^2 e^{-\frac{\pi\tilde\lambda^2}{\tau_2}}\right]\quad(3.1)$$

where $q=e^{2i\pi\tau}$

$$\begin{aligned}P_L &= \frac{1}{\sqrt{2T_2U_2}}(n_1+n_2\bar U+m_1\bar T+m_2\bar T\bar U)\\ P_R &= \frac{1}{\sqrt{2T_2U_2}}(n_1+n_2\bar U+m_1 T+m_2 T\bar U)\end{aligned}\quad(3.2)$$

$\tilde\lambda=P_L\tau_2\lambda/\sqrt{2T_2U_2}$, $\bar\theta_1$ is the odd theta-function and $C(\bar q)$ is the partition function of $K3$ in the odd spin structure, that can be shown to depend only on $\bar q$. In particular, $C(\bar q)$ has the following expansion [3]:

$$C(\bar q)=\bar q^{-5/6}(1-244\,\bar q+...)\quad(3.3)$$

where the first factor accounts for the tachyon and 244 is the number of hypermultiplets of the model. In order to show more explicitly the T_2 dependence of $F_H(\lambda)$ rescale $\tau_2\to T_2\tau_2/2$; then, bringing the limit inside the integral and expanding in $\bar q$, the non-vanishing result will be given by the $\bar q^0$ coefficient of the expansion:

$$F_H(\lambda,U)=\lim_{T_2\to\infty}F_H(\lambda,T,U)=\frac{\lambda^2}{2\pi^2}\int_0^\infty\frac{d\tau_2}{\tau_2}\sum_{n_1,n_2}e^{-\frac{\pi\tau_2|P|^2}{2U_2}}\left[240\frac{d^2}{d\lambda^2}\left(\frac{\lambda\pi}{\sin\bar\lambda}\right)^2+16\pi^2\right]\quad(3.4)$$

where $P\equiv n_1+n_2\bar U$ and $\bar\lambda=\lambda\pi\tau_2P/4U_2$. The second term in square brackets comes from the tachyon contribution together with the linear term in $\bar q$ deriving from the expansion of the eta and theta-function and gives a contribution only to F_1[4]. We will see in the next section how this amplitude is reproduced in type I.

[4]Note that F_1 should be evaluated by a three point function [9] but, as showed by [3], the computation we have summarized here gives the right result even for this case. The same will happen in Type I theory.

4. Computation of F_g in the type I model

We consider here the same amplitude involving two gravitons and $2g-2$ graviphotons; the graviton vertex operator is the usual

$$V_g^{\mu\nu}(p) = (\partial X^\mu + ip\cdot\psi\psi^\mu)(\bar\partial X^\nu + ip\cdot\psi\psi^\nu)e^{ip\cdot X} \tag{4.1}$$

while the graviphoton vertex operator is obtained applying the two supersymmetric charges to V_g:

$$V_\gamma(p) = (Q_1^{(L)} + Q_1^{(R)})(Q_2^{(L)} + Q_2^{(R)})V_g(p) \tag{4.2}$$

where

$$\begin{aligned} Q_{\alpha,1}^{(L)} + Q_{\alpha,1}^{(R)} &= \oint dz e^{-\frac{\phi}{2}} S_\alpha \Sigma e^{i\frac{H_5}{2}}(z) + \oint d\bar z e^{-\frac{\tilde\phi}{2}} \tilde S_\alpha \tilde\Sigma e^{i\frac{\tilde H_5}{2}}(\bar z) \\ Q_{\alpha,2}^{(L)} + Q_{\alpha,2}^{(R)} &= \oint dz e^{-\frac{\phi}{2}} S_\alpha \bar\Sigma e^{i\frac{H_5}{2}}(z) + \oint d\bar z e^{-\frac{\tilde\phi}{2}} \tilde S_\alpha \bar{\tilde\Sigma} e^{i\frac{\tilde H_5}{2}}(\bar z) \end{aligned} \tag{4.3}$$

and $e^{-\frac{\phi}{2}}$, $e^{i\frac{H_5}{2}}$ are the bosonization of the superghosts and of the complex fermion associated to the internal torus respectively, S_α is the space-time spin field operator and Σ and its complex conjugate are the $K3$ internal spin field operators; bosonizing the U(1) Cartan current in the SU(2) algebra of the internal $N=(4,4)$ SCFT as $J_3 = i\sqrt{2}H$, Σ can be written as $\Sigma = e^{i\frac{\sqrt{2}}{2}H}$. The same of course for the right-moving sector. The 1-loop amplitude involves a sum over the torus, Klein bottle, annulus and Möbius strip surfaces. We may now use the results of ref.[10], in order to compute the boson and fermion propagators on all the surfaces starting from those on the torus, by the method of images; similarly to what found in [11] (and following the same notation), it is then possible to see that after having extracted the right dependence on momenta and summed over the spin structures over all the surfaces, the F_g couplings are given by the following amplitude in the odd spin structure:

$$F_{g,I}(U) = \frac{1}{(g!)^2\pi^2} \sum_{\substack{\alpha=\mathrm{T,K}\\ \mathrm{M,A}}} \int [dM]_\alpha\, C_\alpha([t]) \sum_{n_1,n_2} e^{-\frac{\pi[t]|P|^2}{U_2\sqrt{G}}} \left(\frac{P}{2U_2\sqrt{G}}\right)^{2g-2} \langle V_g^+ V_g^- \prod_{i=1}^{g-1}\prod_{j=1}^{g-1}$$

$$\left[\int d^2x_i (Z_1^+(\partial+\bar\partial)Z_2^+ + (\psi_1^+ - \tilde\psi_1^+)(\psi_2^+ - \tilde\psi_2^+)\right]\left[\int d^2y_j (Z_2^-(\partial+\bar\partial)Z_1^- + (\psi_1^- - \tilde\psi_1^-)(\psi_2^- - \tilde\psi_2^-)\right]\rangle_\alpha$$

where

$$V_g^{\pm} = \int d^2x (Z^{\pm}_{1,2}\partial Z^{\pm}_{2,1} + \psi_1^{\pm}\psi_2^{\pm})(Z^{\pm}_{1,2}\bar\partial Z^{\pm}_{2,1} + \tilde\psi_1^{\pm}\tilde\psi_2^{\pm}) \tag{4.4}$$

$[dM]_\alpha$ denotes the measure of the moduli integration for each surface α with $[t]$ the corresponding coordinate, $C_\alpha([t])$ is the partition function in the odd spin structure of the internal sector and $P = n_1 + n_2\bar U$ are the discrete momenta corresponding to the T^2 torus. Define now

$$F_I(\lambda, U) \equiv \sum_{g=1}^{\infty} g^2 \lambda^{2g} F_{g,I}(U) \tag{4.5}$$

Exponentiating we obtain

$$F_I(\lambda, U) = \frac{\lambda^2}{\pi^2} \sum_{\substack{\alpha=\mathrm{T,K} \\ \mathrm{M,A}}} \int [dM]_\alpha \, C_\alpha([t]) \sum_{n_1,n_2} e^{-\frac{\pi t |P|^2}{U_2\sqrt{G}}} \langle e^{-S_0 + \tilde\lambda S} V_g^+ V_g^- \rangle_\alpha \tag{4.6}$$

where $\tilde\lambda = \lambda t P/\sqrt{2U_2}G^{1/4}$, S_0 is the free action for the space-time bosons and fermions and

$$S = \int \frac{d^2x}{t} [Z_1^+(\partial+\bar\partial)Z_2^+ + (\psi_1^+ - \tilde\psi_1^+)(\psi_2^+ - \tilde\psi_2^+) + Z_2^-(\partial+\bar\partial)Z_1^- + (\psi_1^- - \tilde\psi_1^-)(\psi_2^- - \tilde\psi_2^-)] \tag{4.7}$$

Because of the four zero modes $\psi_1^{\pm} = \tilde\psi_1^{\pm}$ =const., $\psi_2^{\pm} = \tilde\psi_2^{\pm}$ =const. of the new action, it is easy to check that

$$\langle e^{-S_0 + \tilde\lambda S} V_g^+ V_g^- \rangle_\alpha = \frac{t^2}{2} \frac{d^2}{d\tilde\lambda^2} \langle e^{-S_0 + \tilde\lambda S} \rangle_\alpha \tag{4.8}$$

The amplitude is then reduced to the evaluation of determinants of space-time bosons and fermions; before computing them, however, note that $C([t])$ is actually an index on all the surfaces. In order to see this, it is better to go to the operatorial formalism. For the torus and annulus, $C(t)$ coincides with the Witten index [12] $\mathrm{Tr}_{\mathrm{R-R}}(-)^{F_L+F_R} q^{L_0}\bar q^{\bar L_0}$ and $\mathrm{Tr}_{\mathrm{R}}(-)^F q^{L_0}$. For the Möbius strip and Klein bottle, $C_\alpha(t)$ is respectively $\mathrm{Tr}_{\mathrm{R}}\Omega(-)^F q^{L_0}$ and $\mathrm{Tr}_{\mathrm{R-R}}\Omega q^{L_0}\bar q^{\bar L_0}$. It is easy to see that these are still indices. In the Möbius strip Ω will act simply by multiplying each multiplet by a common eigenvalue, while in the Klein bottle it is possible to check that each multiplet entering in the evaluation of the trace has equal number of states with opposite eigenvalues of Ω. Since the $C_\alpha([t])$ are all indices, their values are invariant for small perturbations of the theory and then it is possible to

compute them directly from the spectrum of the free theory considered in [6]. By a simple counting in the closed string spectrum, we easily derive that $C_{\rm K} = 0$ and $C_{\rm T} = 8$ [5]. Since in the torus the fermion and boson determinants cancel, leaving a λ-independent constant, we have a non-vanishing contribution only for the coupling $F_{1,I}$ by the following term:

$$F_{1,I}^{\rm torus} = \frac{1}{\pi^2} \int d^2\tau \, C_{\rm T} \sum_{n_1,n_2} e^{-\frac{\pi\tau_2 |P|^2}{U_2\sqrt{G}}} \langle V_g^+ V_g^- \rangle_{\rm T} \tag{4.9}$$

The only non-vanishing contribution comes when we take all the eight fermions of the two graviton vertex operators to soak up the eight zero-modes present in the odd spin structure of the torus. The result gives simply:

$$F_{1,I}^{\rm torus} = 4 \int \frac{d^2\tau}{\tau_2} \, C_{\rm T} \sum_{n_1,n_2} e^{-\frac{\pi\tau_2 |P|^2}{U_2\sqrt{G}}} \tag{4.10}$$

Rescaling $\tau_2 \rightarrow \sqrt{G}\,\tau_2/2$, we obtain, for large $\sqrt{G}$:

$$F_{1,I}^{\rm torus} = 32 \int_0^\infty \frac{d\tau_2}{\tau_2} \sum_{n_1,n_2} e^{-\frac{\pi\tau_2 |P|^2}{2U_2}} \tag{4.11}$$

The remaining contribution for the F_g's comes from the annulus and Möbius strip. The corresponding determinants give equal contribution, with the following result:

$$\langle e^{-S_0 + \tilde\lambda S} \rangle_{\rm M} = \langle e^{-S_0 + \tilde\lambda S} \rangle_{\rm A} = \frac{1}{t^3} \prod_{m=1}^\infty \left(1 - \frac{\tilde\lambda^2}{m^2}\right)^{-2} = \frac{1}{t^3} \left(\frac{\tilde\lambda\pi}{\sin\tilde\lambda\pi}\right)^2 \tag{4.12}$$

Looking to the open string spectrum of [6], it turns out that $C_{\rm A} + C_{\rm M} = 32^2 - 2\cdot 32 = 4\cdot 240$. Putting everything together and rescaling $t \rightarrow \sqrt{G}\,t/2$, we finally obtain:

$$F_I(\lambda, U) = \frac{2\lambda^2}{\pi^2} \int_0^\infty \frac{dt}{t} \sum_{n_1,n_2} e^{-\frac{\pi t |P|^2}{2U_2}} \left[240 \frac{d^2}{d\lambda^2} \left(\frac{\lambda\pi}{\sin\bar\lambda}\right)^2 + 16\pi^2 \right] \tag{4.13}$$

where $\bar\lambda = \lambda\pi t P/4U_2$, reproducing, up to a constant, the generating function $F_H(\lambda, U)$, according to the type I-heterotic duality. Note also that only $N = 2$ BPS states give contributions to the F_g couplings, like in the heterotic case [13].

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[5] Note that the twisted closed states in the orbifold limit are all massive [8].

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